CONCEPTUAL SPACE AND PROJECTIBILITY

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Summary. According to Gärdenfors, the formal property of convexity in a conceptual space can serve as a criterion of projectibility. This theory is here criticized on two grounds: 1) Gärdenfors is inconsistent when applying the criterion to the troublesome Goodman predicate 'grue'; 2) dispositional predicates are typically non-convex and yet projectible. Projectibility is suggested to be neither a matter of logic nor one of symbolism.

Introduction

Spatial representation, in one form or another, is a fruitful tool for the analysis of concept denotations and connotations, as is evident from, *e.g.*, Principal Component Analysis and Projection to Latent Structures in science (Wold *et al.* 1984), Nearest-neighbour and Voronoi categorizations in experimental psychology (Gärdenfors and Holmqvist 1994), and the use of Venn diagrams in class logic. However, it can be questioned whether a conceptual space can really achieve everything that one has wanted it to. Gärdenfors (1990, 1992) proposed that projectible predicates are recognizable by virtue of their formal properties in a conceptual space. A rule differentiating between projectible and non-projectible predicates was presented (subsequently referred to as the convexity rule):

> 'The topological properties of the dimensions now allow us to introduce the notion of a natural property, which we have seen to be a central task for a theory of induction. The definition is simply that a property, i.e. a region of a conceptual space, is natural only if the region is convex. A convex region is characterized by the criterion that for every pair s_1,s_2 of points in the region all points in between s_1 and s_2 are also in the region.'

Gärdenfors (1990) applied the convexity rule to the gruesome projectibility problem of Goodman, *i.e.* the question why observations of green emeralds confirm the hypothesis, 'All emeralds are green', but not the hypothesis, 'All emeralds are grue' ('grue' meaning green before the year 2000 and blue thereafter). 'Grue' was suggested to be a non-projectible predicate because of its spatial representation being non-convex.

Like Gärdenfors I tend to believe that a solution to the projectibility problem requires some idea of natural kind. Unlike the claim made in the convexity rule, however, I do not think that natural kinds can be identified by formal criteria, be they linguistic or otherwise symbolic. In this brief comment I shall try to demonstrate three things: 1) that Gärdenfors is inconsistent in his own argument for the convexity rule, 2) that convexity within a natural conceptual space is not a condition for projectibility, and that 3) ’grue’ may well be a projectible predicate.

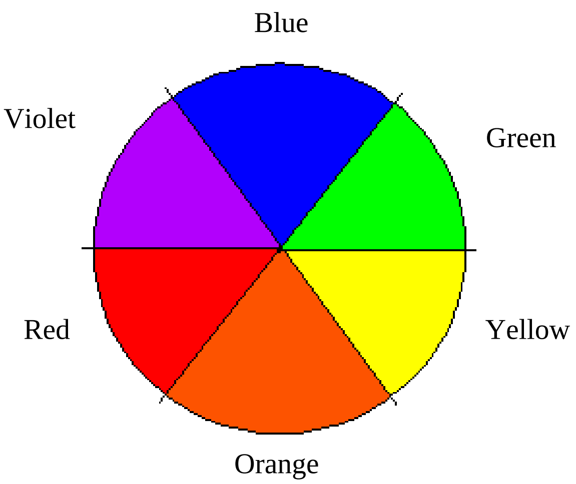

FIG. 1. Standard colour circle

Gärdenfors on 'grue'

The standard colour circle (Fig. 1) is an example of a conceptual space. All of the regions representing various colours are convex and hence the colour predicates are projectible according to the convexity rule. By adding a time dimension to the colour circle, a somewhat more complex conceptual space is obtained in the form of a tube (Fig. 2). Here it can be seen that 'grue' is represented by a non-convex space. Therefore, according to the convexity rule, 'grue' is a non-projectible predicate.

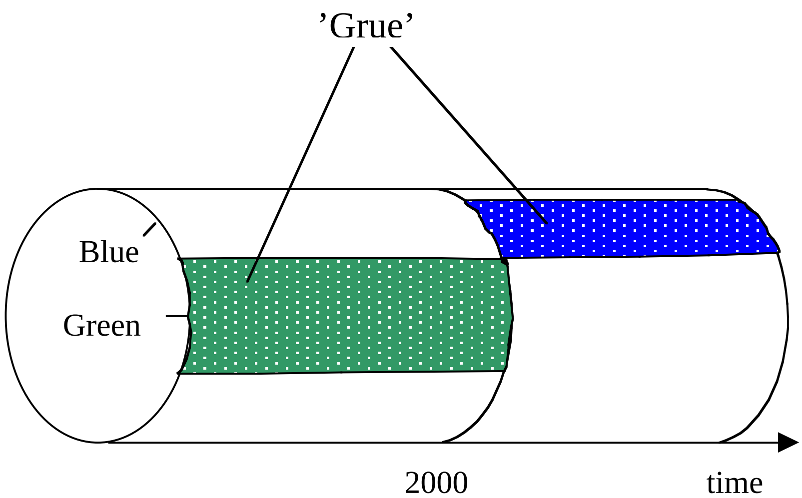

FIG. 2. Spatial representation of 'grue', showing its non-convexity. After Gärdenfors (1990)

However, it turns out that this lack of convexity is in fact not the reason why Gärdenfors (1990) considers ’grue’ an unnatural and non-projectible predicate. As he himself remarks in passing,

> ’[…] it is not difficult to construct a conceptual space where ’grue’ would correspond to a convex region and thus be a natural property in that space.’

Such a space is not depicted in Gärdenfors’ paper, but Fig. 3 is an obvious possibility. The convexity of ’grue’ in that space is gained by allowing ’grue’ and ’bleen’ to be among the a priori dimensions from which the space is constructed. Fig. 3 can be understood as a spatial illustration of Goodman’s classical answer to the objection that ’grue’ and ’bleen’ are more complex predicates than ’green’ and ’blue’: if one starts with ’grue’ and ’bleen’, then ’green’ and ’blue’ become the more complex predicates, requiring the dimension of time for their definition. Thus, in terms of Gärdenfors’ conceptual space and convexity symbolisms, Fig. 3 merely iterates Goodman’s gruesome projectibility problem by raising the question why, precisely, convexity in Fig. 3 does not afford projectibility if convexity in Fig. 2 does.

It seems to me that Gärdenfors' answer rather inconsistently has nothing to do with convexity at all. In brief, the arguments given are instead that not all conceptual spaces are good enough, that the legitimate ones are based on natural quality dimensions, and that 'Goodman-type predicates will in no case count as natural.' Having conceded, although in passing, that Goodman-type predicates

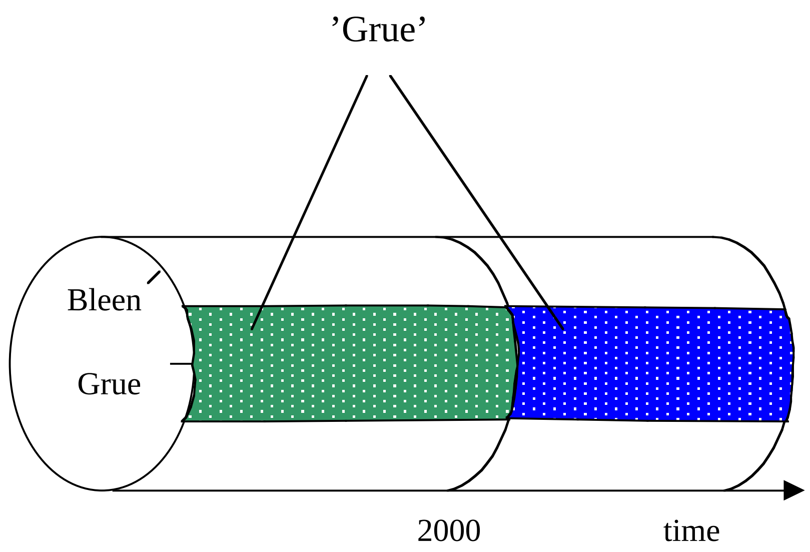

FIG. 3. Spatial representation of 'grue', showing its convexity.

can readily be given a convex representation in some allegedly illegitimate conceptual spaces, Gärdenfors (1990) avoids the circular argument that such predicates are prohibited as space-founding dimensions because they are non-convex. Thus, 'grue' is after all not rejected for the reason that it is non-convex, neither as a region within a space nor as a dimension underlying a higher-order space. So, the question remains: why is 'grue' rejected? And why bother about convexity?

Someone could perhaps be tempted to argue that the quality dimensions of the non-standard colour circle used to construct the conceptual space in Fig. 3 are in themselves non-convex, unlike the convex colour regions in Fig. 1. Such an argument would be mistaken. Fig. 4 shows the complete non-standard colour circle. There is, of course, nothing in the spatial representation as such that would make the sectors in Fig. 4 less convex than those in Fig. 1. Morover, the topological relations within the circle are time-invariant. For example, in Fig. 4 ’grue’ is located between ’bleen’ and ’yellet’. Translating this fact, by means of the time variable, to ordinary colour concepts, one obtains that before 2000 ’green’ is located between ’blue’ and ’yellow’, and that after 2000 ’blue’ is located between ’green’ and ’violet’, as is also the case before 2000. All the non-standard colour-related qualities in Fig. 4 show a similar constancy in maintaining their respective neighbours when translated into time-variant ordinary colours. Finally, although self-evident it is important in the present context that for every pair s_1s_2 of points in any region in Fig. 4 (*e.g.* yellet), all points between s_1 and s_2 are also in that region, both before 2000 (when the exemplifying region corresponds to yellow) and after (when it corresponds to violet).

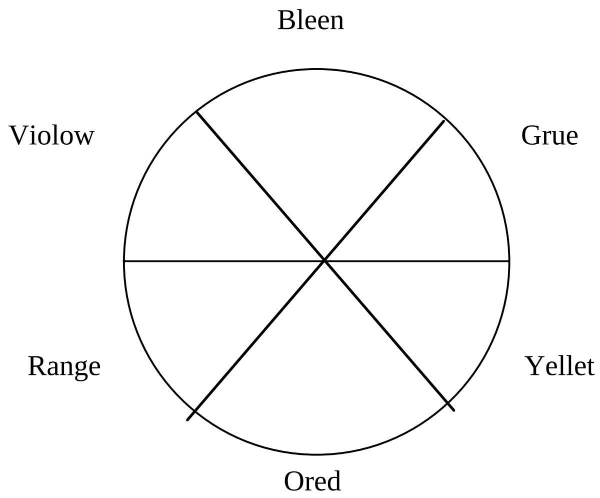

FIG. 4. Non-standard colour circle

Projectibility does not require natural convexity

The peculiar aspect of ’grue’ and similar predicates is not, as if often thought, that they combine a more elementary quality with time. It is not unnatural for a predicate to signify that some quality changes. That characteristic is shared by perfectly normal and projectible predicates expressing a disposition or some

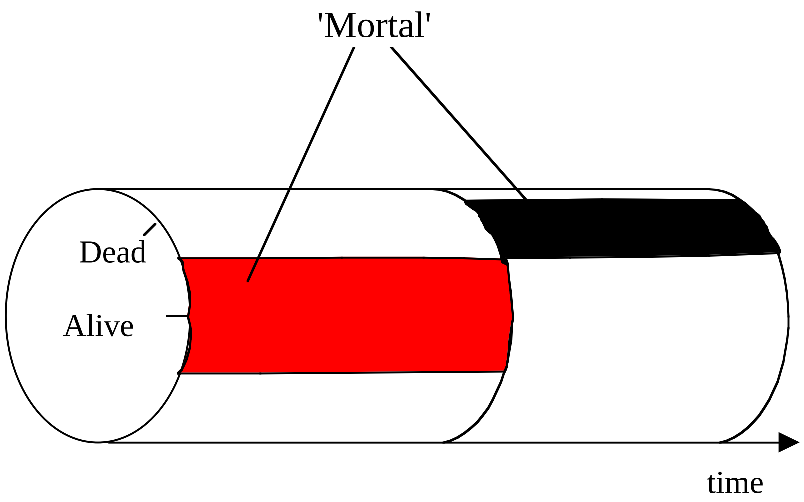

FIG. 5. Spatial representation of 'mortal', showing its non-convexity.

more definite kind of alteration with time. Consider, for example, the following two well-tested empirical generalizations: 1) all stock prices are instable; 2) all men are mortal. The instabilities of stock prices and human lives can be represented in conceptual spaces employing some quality measure and time as orthogonal dimensions. In either case, non-convex trajectories or regions in the space would symbolize the one thing that is constant and inductively certain

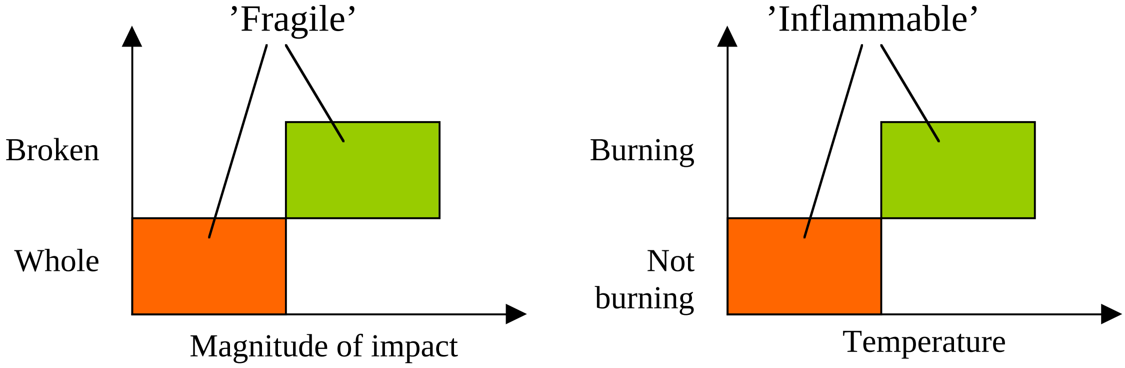

FIG. 6. Conceptual spaces showing non-convexity of the projectible predicates, ’fragile’ and ’inflammable’

about stock markets and lives, *i.e.* that they are inconstant. In the example given in Fig. 5, the rather crude qualities 'alive' and 'dead' can, of course, be further differentiated, much like the crude concepts 'blue' and 'green' that cover various shades between violet and yellow.

That non-convexity clearly does not exclude projectibility is also demonstrated by physical phase transition diagrams not explicitly involving time. Such diagrams can represent trivial everyday phenomena – *e.g.* the breaking of glass, the ignition of fire, the forming of ice, the boiling of water – which certainly are natural enough to fall within the realm of any reasonable theory of projectibility and induction. The dispositional quality terms for the breaking of glass (’fragile’) or the ignition of fire (’inflammable’) seem straightforward enough (Fig. 6).

It is more difficult to find a word that expresses the disposition of water to alternate between its familiar phases ice, fluid water and steam. The reason why such a predicate word does not seem to exist in natural language is probably the mere lack of practical need for it, rather than any logical shortcomings of such a concept formation. If a word were invented, say ’metamorphic’, the predicate would certainly be projectible in spite of the fact that its most obvious spatial representation would depict it as a non-convex function of ’solid’, ’liquid’ and ’vaporous’ on the one hand and temperature on the other.

Confirmation

Quite apart from the problems attending the convexity rule, one may ask why ’grue’ should not after all be considered a projectible predicate, if ’instable’, ’mortal’, ’fragile’ etcetera are. All dispositional predicates tell us that something can be in at least two states, that it is (was) in one of these states at one time point and that it may be (was) in the other state at some other time point. How does one confirm a generalization involving a conventional dispositional predicate? Certainly not by performing observations of only one of the states. For example, to confirm that all men are mortal or that a glass is fragile, it is not sufficient to observe a living man or a whole glass now. Neither does it suffice to observe a corpse or a heap of splintered glass. For any object to count as a confirmatory instance it must be observed in both its states, *i.e.* at no less than two points in time. Is there any compelling argument for treating ’grue’ differently?

The reason why the observation of green emeralds does not confirm that all emeralds are grue is perhaps not that there is something illegitimate about ’grue’, but simply that green is not grue. To confirm that all emeralds are grue, one should observe the grueness of at least one emerald, an act which requires obvservations both before and after 2000. If some such observations will as a matter of fact be made, it would seem natural to say that we are dealing with a normal inductive confirmation of the hypothesis that all emeralds are grue. That projectibility is not primarily a matter of logic is discussed by Gärdenfors (1990). As suggested here, it is not a matter of cognitive symbolisms either. More probably, the peculiarity of ’grue’ simply reflects the empirical content of present-day knowledge about emeralds and other physical objects.

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