# ARE FULL BELIEFS AN ABSTRACTION FROM CREDAL STATES?

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## 1. States of Full Belief, Confirmational Commitments and Credal States

An account of belief change appropriate to providing a systematic understanding of well conducted inquiry presupposes a characterization of changes of belief state that are open to the inquiring agent to make in the course of the inquiry. How the space of belief states is conceived may have a bearing on the account of belief change that may be given. Peter Gärdenfors has always been acutely aware of the importance of this point. In a review of *Decisions and Revisions* (Levi, 1984), he expressed some reservations about my approach and offered an alternative perspective (Gärdenfors, 1987). On this happy occasion, I mean to honor Peter by explaining why I resist his suggestion.

According to a view I developed in (Levi, 1974, 1980), the doxastic state of an inquiring agent at a given time (or in a given situation) has two components: a *state of full belief* and a *confirmational commitment*.

Agent X's confirmational commitment is a function specifying the state of subjective or credal probability judgment (credal state) to which X is committed for each potential state of full belief. It is representable by a function *C*: **K** -> **B** from potential state of fully belief **K** to credal state **B**. Consequently, given the doxastic state as specified by **K** and *C*, the credal state to which X is committed at the given time or in the given situation is uniquely determined. A change in doxastic state is, as a consequence, either a change in state of full belief, a change in confirmational commitment or a change in both.

Gärdenfors notes that I say, "Rational men should not alter their confirmational commitments for revising credal states without good reason." He concludes from this that I regard such change "as something exceptional" (1987, p.748). Gärdenfors forgets that on my view one should not change states of full belief without good reason either. Yet, changes in states of full belief are not "exceptional" on my view. Why should changes in confirmational commitments be?

In the very essay on which he comments, I wrote, "In my opinion, confirmational commitments ought often to be subject to critical review and revision" (1984, p.205). In point of fact, in (Levi, 1980, 1.4-13.10) I suggest an account of some conditions under which this can happen and how it should happen.

Indeed, my motive for distinguishing between confirmational commitments and states of full belief is that doing so allows for changes in probability judgment to take place *independently* of changes in states of full belief and vice versa. It may, of course, turn out that the two components are strongly correlated with one another. But it may turn out otherwise. It is desirable to have a way of representing doxastic states that does not settle this issue in advance.

Gärdenfors's own way of proceeding does not permit this. Gärdenfors suggested that I should have considered a function from credal states (i.e., states of subjective probability judgment) to states of full belief. Indeed, he offered a specific such function. Given a credal state **B**, any proposition assigned probability 1 according to **B** is fully believed in **K**. Although subjective or credal probabilities could change without states of full belief changing, changes in states of full belief must lead to alterations in states of credal probability judgment. Distinguishing between confirmational commitments and states of full beliefs does allow for independent variation of two components one of which controls probability judgment and the other full belief.

There are other advantages of my proposal over Gärdenfors's as well. As Gärdenfors notes, I agree that any proposition in **K** and, hence, fully believed should have probability 1. But I reject the converse he favors. He complains that it is not clear what further properties a proposition must have additional to credal probability 1 in order to be fully believed. He concludes that there is no additional property to seek. He suggests, therefore, that the "best starting point" for identifying such additional properties is to construe a confirmational commitment as a function from **B** to **K** along the lines he suggests.

## 2. Why credal probability 1 at t is not sufficient for full belief at t

There is ample reason for calling sufficiency of probability 1 for full belief into question.

Consider, for example, the task of estimating the mean of some normally distributed random variable. Bayesian statisticians often recommend using a uniform prior probability distribution over the mean. I do not wish to suggest that one should, in such cases, mandate adopting a uniform prior probability for the mean. Indeed, in general, I do not think it is a very good idea. But adopting a uniform prior should not be *forbidden* under all circumstances.

Suppose a situation arises where the uniform distribution is appropriately adopted. Then if we focus on a partition of the real line into interval estimates of equal positive Lebesgue measure, there will be a countable infinity of such estimates covering the point estimates from $-\infty$ to $+\infty$. And each of them will carry 0 finitely additive credal probability. According to Gärdenfors's proposal, the negation of each and every one of these alternatives carries credal probability 1 and should be fully believed. This means that each of the countable infinity of interval estimates carrying 0 credal probability should be fully believed to be false.

This may not seem troublesome if "full belief" is taken to be synonymous with "judged probable to degree 1". If we adopt this linguistic practice, we can say *pace* G.E. Moore, "I believe that h but h might be false". We can acknowledge that the true value of the mean might be found in any of the intervals in the partition while declaring ourselves full believers that the truth is not to be found in that interval.

If that is the practice followed, we shall need a distinction between two kinds of full belief. In claiming that probability 1 is a necessary but not a sufficient condition for full belief, I was implying that we need a distinction between two kinds of probability 1 judgment one of which is full belief. I will continue to use full belief to mark the kind that precludes the serious possibility that the negation of a full belief is true and recognize the other kind of "almost certain" beliefs to consist of those propositions judged possibly false while carrying probability 1. Adopting this usage, the Moorean sentence does indeed become incoherent.

My practice presupposes, however, that there are, indeed, two distinct kinds of probability 1 judgment - where probability is subjective or credal probability. Gärdenfors may wish to challenge that distinction. As I understand the challenge, I must respond by offering a rationale for recognizing two distinct kinds of (subjective) probability 1 judgment.

In a game where a coin is to be tossed until it lands heads for the first time, the probability that the game will stop after a finite number of tosses is 1. Yet, the game might not stop. Contrast this with the judgment that the probability is 1 that either the game will stop at the nth toss or stop after some number n of tosses or will never stop.

Or consider the problem of estimating the value of the mean of a normal distribution starting with a uniform prior over the range of values for the mean on the real line. As before, we take any partition of the real line into segments of positive finite Lebesgue measure. Probability 1 is assigned to the judgment that the true value of the mean is a real valued quantity ranging somewhere between $-\infty$ and $+\infty$. It is also assigned to each of the countably infinite propositions implying that the true value of the mean is not in a given one of the cells in the partition.

There are three things we can do here:

(1) We can say that probability 1 is used in the same sense of full belief so that the set of full beliefs is inconsistent. However, full belief does not, as it does for Moore and me, rule out the possibility of being false.

(2) We can say that probability 1 is used in the same sense throughout so that inconsistency emerges as in case (1). However, full belief rules out possibility of being false.

(3) We can say that there is a distinction between probability 1 assigned to the judgment that the true value lies on the real line and probability 1 assigned to the judgment that the true value does not lurk in any specific finite interval.

The first two variants lead to infinitary versions of the infamous lottery paradox.[1] Version (1) leaves a bolt hole where one can escape from outright incoherence by acknowledging that that each and every one of the beliefs might be false. Consequently, it is coherent to admit at one of the beliefs is false. But consider any finite version of the lottery paradox. In that case, we do rule out as impossible the judgment that no ticket will be drawn. As the number n of tickets goes to infinity, we continue to rule it out.

The second variant is truly incoherent. Not only is the set of full beliefs inconsistent but the agent is committed to denying the possibility that any of them is false. Moreover, if we consider the finite lottery, the prospect of a given ticket winning carries positive probability and remains possible as n goes to infinity. The limit of this probability is 0 but in the limit the prospect remains a possibility. So aside from the incoherence, the second version is no more plausible than the first.

This leaves the third version that concedes the distinction between probability 1 as full belief or absolute certainty and probability 1 as almost certainty.

As just noted, I understand probability 1 (in the *standard* real numbers) to be necessary but not sufficient for full belief. What is the "extra condition" required here? The example of estimating the mean of a normal distribution just considered makes this clear. When facing a problem calling for inquiry, we explore a range of potential answers. One demand we impose on potential answers to a question is that they not only be logically consistent and thus logical possibilities but that be what I call serious possibilities that are not already ruled out by what is already settled. At the beginning of the task of estimation, we do not rule out any one of the values of the mean lying on the real line although we do rule out that the mean is an n dimensional vector for $n \geq 2$. We have assumed more than logic or pure mathematics would allow and we fully believe it in the sense that we take our full beliefs to define the space of possibilities over which we deliberate. This space is the space of serious or doxastic or epistemic possibilities for the inquiring agent. *Pace* Gärdenfors the full beliefs cannot be derived from the state of probability judgment just by determining which propositions carry probability 1.

[1]The lottery paradox is associated with two distinct issues. It can be understood as a consequence of a prescriptive principle of rational belief change. If one begins with full belief that the lottery will be run, you should expand your state of full belief by adding the information that ticket i will lose for each i from 1 to n in the finite case and for each natural number in the infinite case. Or it can be understood as a constraint on what the agent ought to fully believe at a given time. I am focusing on the case where it is a constraint on the attitudes of an agent at a given time or from a single point of view. This is especially relevant in the context of the infinite lottery where it is tempting to think of probability 1 at time t as sufficient for full belief at time t and not merely for expanding to full belief at t'.

Perhaps, however, full belief as I understand it, can be derived from the credal state in another way.

One might, perhaps, appeal to the conception of the "support" of a probability measure. Suppose the logically possible values of the mean are all real values from minus infinity to plus infinity. Suppose, however, that the support for the probability distribution is the interval from -1 to +1. What this means is that the *closed* interval in question is the smallest closed set whose probability is 1. Someone might think of proposing that the set of seriously possible point estimates consists of all points in the support of the probability distribution.

This suggestion cannot be satisfactory. In the first place, the support of a distribution is relative to a topology on the real line whereas the domain of seriously possible hypotheses is not. In addition, a set of point estimates carrying total probability 0 can be deleted from the support as not being seriously possible. So there is a difference between belonging to the support and being seriously possible.

A better idea might be to represent the state of probability judgment by a conditional probability measure *Q*(h/e) well defined for every h in the language and for every e expressing a seriously possible proposition. If the conditional probability is taken to be primitive, the seriously possible can be defined as the set *E* of propositions such that *Q*(h/e) is defined for e $\in$ *E*. The state of full belief **K** is represented by the set of sentences whose negations are not seriously possible.

This proposal would be acceptable for those sympathetic with De Finetti's use of conditional probability to represent credal states. But if one sought to represent credal states with the aid of countably additive unconditional probability measures after the fashion of Kolmogorov, conditional probability should be understood differently. Given a probability space consisting of a set $\Omega$ of maximally specific propositions (in the context), a field *F* or $\sigma$ - algebra of subsets of $\Omega$ and an (unconditional) probability measure *P* over *F*, consider some sub $\sigma$-algebra *G* of *F*. Relative to such *G* it is possible to define for fixed h a function *P*(h||*G*)(g) defined over all g in *G* in one of infinitely many ways (all of which are called "versions") that all yield the same value except for a set of elements of *G* carrying probability 0. Moreover, for g in *G*, *P*(h||*G*)(g) obeys the multiplication theorem and satisfies the condition that *P*(h) is the integral of the function *P*(h||*G*)(g) for g's *G* that partition $\Omega$. Finally if we fix *G* and g in *G* and let h vary, *P*(h||*G*)(g) satisfies the requirements of the calculus of countably additive probability with probability 1.

It is clear that this notion of conditional probability is intended to cover cases where the unconditional probabilities of propositions or events are 0 and yet conditional probabilities are defined relative to such events. But it yields such an extension at best "with probability 1" and even then only with respect to a sub $\sigma$-algebra *G* that does not coincide in general with the space of serious possibilities. The Kolmogorovian practice of relativizing conditional probability to subfields and resting content with almost sure agreement between the results of using Radon Nikodym derivatives and conditional probability does not yield what the project suggested by Gärdenfors requires.

Kolmogorovians cannot endorse the De Finetti approach. De Finetti's strategy requires abandoning the countable additivity requirement. If countable additivity is abandoned, a state of full belief can be represented by a state of credal probability judgment characterized by a set of conditional probability functions.

This is not Gärdenfors's suggestion. According to his proposal, a state of full belief is represented by a credal state represented by an unconditional probability. Conditional probability is defined only for conditions with positive probability. So claiming that e is seriously possible if and only if the conditional probability *p*(x/e) is well defined reduces to equating serious possibility with carrying positive probability. Full belief is equated with probability 1 - a view we have seen to be unsatisfactory. Even so, if De Finetti's approach is adopted and credal states are characterized by conditional probability functions, there is a way to derive states of full belief from credal states.

## 3. Truth Value

My concession to Gärdenfors does not mean that I think that studying doxastic states characterized by conditional probability functions or sets of such functions is a sensible approach to adopt. Philosophical and technical points guide my reluctance.

Potential states of full belief are free of error or are erroneous. Moreover, an inquiring agent X is committed to judging X's current state of full belief and all its consequences to be free of error. That is to say, X is committed to judging all X's current full beliefs to be true. In addition, if X is concerned to avoid error in changing X's state of full belief, X should avoid deliberately shifting from X's current state to any potential state that implies its complement. Expanding the current belief state **K** by adding some new item of information not implied by **K** to it may or may not be acceptable. Some risk of error will be incurred. To be warranted, the information promised must be sufficiently valuable to justify incurring the risk.

Probability judgments that are not also full beliefs are neither true nor false. (This is so even in cases where X is "almost certain" that h is true - i.e., assigns credal probability 1 to h but judges it a serious possibility that h is false.) A state of belief as Gärdenfors suggests I should understand it has a truth-value bearing part and a part that lacks truth-values. In examining belief change, I contend that we consider decomposing a state of belief of the sort Gärdenfors suggests we use into two constituents: one for which it makes sense to urge avoidance of error and another where it does not.

Gärdenfors's account of belief change does not address the issue of whether avoidance of error in the sense of avoiding the importation of false beliefs is a desideratum in inquiry and there is some explicit indication that he is not at all concerned with this matter. To the extent that he adopts a view that is in this way indifferent to the concern to avoid error, the question as to whether probability judgments do or do not carry truth-values may not be highly significant for him. But anyone who thinks that a concern to avoid importing false beliefs is a critical component of the aims of well conducted inquiry should acknowledge that there is an important difference in the conditions under which changes in full belief are justified and changes in probability judgment are justified. Such a person should find it an advantage to treat changes in full belief and in probability as two separable factors whose independence from one another could be explored without begging questions. And those, like Gärdenfors, who may not attach much significance to the concern to avoid error, ought not to beg the question against those who do by assuming without argument that full belief and probability are inseparable.

Decomposing doxastic states into states of full belief and confirmational commitments is a way of allowing for the independent variation of states of probability judgment and states of full belief. Potential confirmational commitments are neither true nor false. It makes no sense to evaluate the risk of error incurred by adopting such a commitment. Perhaps, a concern with avoiding error may be relevant to choosing between rival confirmational commitments; but the concern is not with avoiding error in the confirmational commitments selected but in the potential states of full belief that may be adopted as a consequence. Thus, it may make sense to explore changes in states of full belief separately from changes in states of probability judgment. For this purpose, it is preferable to consider confirmational commitments as the states of probability judgment to be subjected to change rather than the credal states that are the values of such commitments.

Focusing on changes in confirmational commitments might appear to neglect those modes of change in probability judgment that appear to involve no change in the state of full belief. Gärdenfors suggested in particular that my approach would preclude consideration of Jeffrey updating. This is not true. Jeffrey updating involves changing probability judgment by changing the confirmational commitment while leaving the state of full belief unaltered. It is not the only kind of change of this sort one might imagine but it is one kind. Although I think that Jeffrey updating is not to be recommended as a way to revise probability judgment, my reservations with this idea are not a consequence of factoring doxastic states into states of full belief and confirmational commitments. The objection I have to this and to other such approaches can be usefully elaborated with the aid of the factorization into states of full belief and confirmational commitments as I shall try to explain later. To see this, we should review the properties of confirmational commitments and states of full belief.

## 4.Prescriptive Comparative Statics

X's state of full belief at t is constituted by X's firm convictions and the commitments to full belief generated by such commitments. Insofar as such a state **K** can be represented in a regimented language L, a deductively closed set K of sentences in L may represent it. If a sentence h in L is in K, X is committed to judging it true in L and to ruling out the logical possibility that h is false as a serious possibility. In this sense, X is committed to fully believing that h. X may not fulfill X's commitments due to failures of memory, computational capacity or due to emotional disturbances. X still has the commitments in the sense that X has the obligation as a rational agent to undertake steps to improve X's performance.

Thus, Jones is in a state of full belief at t that commits Jones to full belief that Bill is the father of Joe. Moreover, Jones fulfills the commitment in the sense that if he were asked whether Bill is the father of Joe, Jones would answer affirmatively, Jones would include Joe in a party where all of Bill's children are invited and the like. Jones, in this sense, has many of the dispositions that constitute fully believing that Bill is the father of Joe. To repeat, Jones is in a state of full belief or a state of doxastic commitment to full beliefs including full belief that Bill is the father of Joe. Jones at least partially fulfills the commitment to full belief that Bill is the father of Joe by having some of the doxastic dispositions to linguistic and other behavior associated with full belief that Bill is the father of Joe. Jones may also manifest some of these doxastic dispositions associated with full belief that Bill is the father of Joe such as actually offering an affirmative answer to a question put to him or thinking to himself that Bill is the father of Joe. Jones may be said to believe that Bill is the father of Joe in three distinct senses:

(1) Jones is in a state of full belief that commits or obligates Jones to have certain doxastic dispositions associated with believing that Bill is the father of Joe.

(2) Jones may partially fulfill that commitment by have some of these doxastic dispositions.

(3) Jones may have actually manifested some of these doxastic dispositions.

Suppose that some time t', Jones's state of full belief is modified by his adding the information that Joe is married to Sue. Presumably note only have Jones's doxastic commitments changed but so have his dispositions - to the extent that he has doxastic dispositions associated with full belief that Joe is married to Sue. He may or may not have manifested these dispositions. More to the point, Jones may have failed to fulfill further doxastic commitments he incurred at t'. Given Jones's initial state of full belief and the new information obtained at t', Jones is committed to fully believing that Bill is the father-in-law of Sue. But Jones may not have figured this out at t' and may not have acquired any of the appropriate dispositions.

Suppose that t'', Jones does recognize the Bill is Sue's father-in-law. If this is all that has happened, Jones has not changed any of his doxastic commitments. His state of full belief is the same as it was at t'. The extent to which he has fulfilled his doxastic commitments has altered and so too may his manifestations of his newly acquired dispositions.

Thus, when talking about belief changes and, even about changes in full belief, we should distinguish between changes in states of full belief, changes in doxastic dispositions that fulfill these commitments and manifestations of such dispositions.

Those who embrace a naturalistic view of the attitudes tend to focus on changes in attitudes that explain and predict linguistic and other behavior. I do not think that belief-desire models do very well in predicting or explaining behavior. That is because the models we have are models of rational behavior that have only limited applicability as predictive or explanatory models. Such models are important to us because they serve as standards by means of which we determine what is needed to improve our behavior as deliberating agents. The belief state agent X is in at a given time is a model that would accurately explain and predict X's behavior were X to fulfill his commitments to full belief (and his other attitudinal commitments). That is to say, it is a model that X would satisfy were X in a state of doxastic equilibrium.

No one is in perfect doxastic equilibrium. Our rationality is bounded by limitations of computational capacity, memory and emotional frailty. But sometimes we seek to improve our performances. By comparing actual performance with the equilibrium to which X is committed, we have some idea of what needs to be done to improve X's performance in fulfilling X's doxastic and more generally rational obligations.

Changes in states of full belief are analogous, therefore, to changes in equilibrium states as studied in classical thermodynamics or in neoclassical economic theories of consumer demand. A systematic investigation of such changes is an exercise in "comparative statics". B.Ellis (1979) and I (Levi, 1970, 1980) have emphasized this point for some time. Gärdenfors (1988),has echoed similar sentiments. Gärdenfors, like Ellis, has tended to think of doxastic equilibrium as playing an explanatory role that I tend to think it can perform only very poorly. That is why, for me, states of doxastic equilibrium are states of doxastic commitment where the inquirer is under some obligation to fulfill the undertakings made when requested and when he or she cannot do so to seek ways and means to improve the ability to do so.

A theory of belief change as a comparative statical theory is prescriptive. Changes in belief states are represented in <u>L</u> by changes in sets of sentences closed under deductive consequence representing X's commitments to full belief before and after the change is instituted. Such a change in commitment does not imply that X's commitments are fulfilled either before or after the change in commitment. Nor is any account provided concerning the psychological path that should be followed in implementing the change and even partially fulfilling the commitments before and afterwards. A recommendation is made concerning the equilibrium state of full belief one should adopt given the initial doxastic commitment without specific recommendation of the details the process to be used in incurring the commitment to attaining that equilibrium or partially fulfilling the obligations thus incurred.

Returning then to the representation of X's state of full belief in L, the deductively closed set K in L may not completely represent X's state of full belief. The limitation is not due to the absence of doxastic or modal operators in L. X's commitments to judgments of serious possibility and impossibility may be represented in a metalanguage ML containing L. X's doxastic commitments expressible in ML are uniquely determined by X's doxastic commitments expressible in L and vice versa. So in studying changes in states of full belief we may ignore the doxastic and modal operators.

The limitation imposed by using L concerns the range of ideas that X might entertain in contemplating changes in X's current state of full belief to a new one. To suppose that any regimented language L or natural language at a given time has the resources to represent the conceptual resources available to X is an assumption we should not make without better warrant than I think any of us has.

In any case, we shall represent X's state of full belief at a given time by a deductively closed theory or corpus K in a suitably regimented language L.

Insofar as changes in X's state of full belief are representable in L, they are representable as sequences of expansions and contractions. (Levi, 1974, 1980 and 1991). Alchourrón, Gärdenfors and Makinson (1985) made a fundamental contribution to the understanding of contraction that has been elaborated and criticized in various ways by many authors. I shall take for granted some familiarity with the AGM account of expansion, contraction and what, to my way of thinking but not according to Gärdenfors, is the derivative notion of AGM revision.[2]

## 5. Confirmational Commitments

The prescriptive comparative statical approach I am adopting to the study of changes in full belief applies *mutatis mutandis* to exploring changes in probability judgment and, indeed, to changes in value judgment as well (a topic that I shall not discuss here). A confirmational commitment will be represented by a function from potential corpora in L to states of credal probability judgment or *credal states*..

X's corpus K at a given time introduces a distinction between sentences in L that are seriously possible at that time and sentences that are impossible. It is seriously possible that h according to X at t (or according to X's corpus K at t) if and only if h is consistent with K. Otherwise h is not serious possible (or is impossible).

[2] AGM revision plays only a marginal role in any plausible account of genuine belief change (Levi, 1991, Hansson (1991). However, AGM revision or the modification of it I call Ramsey revision is of fundamental importance in providing an "epistemic" account of conditionals. (Levi, 1996.)

X's credal state **B** at t relative to K is representable by a set of real valued functions *Q*(x/y) called the *permissible Q-functions according to B*.[3] Each such function is defined for every x and y such that y is consistent with K and the truth value of x is not under X's control at t unless the truth value of y also is.[4]

*Condition of Credal Consistency*: **B** is nonempty If and only if K is consistent.

Condition of *Credal Coherence*: Each *permissible Q-function according to* ***B*** is a finitely additive and normalized probability in L relative to K.[5]

*Condition of Credal Convexity*: Let $\mathbf{B}_y$ be the set of functions in **B** restricted to conditional probabilities on y. For every y for which conditional probability is defined, $\mathbf{B}_y$ is convex. That is to say if *Q*(x/y) and *Q*'(x/y) are in the set so is every weighted average α*Q*(x/y) + (1-α)*Q*(x/y) where 0 < α < 1.

A confirmational commitment for L is a function *C*: *K* -> *B* from the set *K* of potential corpora in L to credal states *B* in L. *C*(K) is a credal state **B** relative to K satisfying credal consistency, coherence and convexity.

Let x be consistent with K and $K^+_x$ be the expansion of K by adding x and forming the deductive closure.

$\mathbf{B}^+_x$ is the *conditionalization* of **B** relative to K and $K^+_x$ if and only if for every permissible *Q*-function *Q* according to **B**, there is a permissible *Q*-function $Q^+_x$ according to $\mathbf{B}^+_x$ such that $Q^+_x(y/z) = Q(y/z \wedge x)$ and for every permissible *Q*-function $Q^+_x$ according to $\mathbf{B}^+_x$ there is a permissible *Q*-function *Q* according to **B** such that $Q^+_x(y/z) = Q(y/z \wedge x)$.

A *quasi - Bayesian* confirmational commitment is one that satisfies the following condition:

*Confirmational Conditionalization*: $C(K^+_x)$ is the conditionalization of *C*(K) relative to K and $K^+_x$.

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[3]Gärdenfors and Sahlin (1982) independently developed a characterization of credal states utilizing a representation by means of sets of probability distributions.

[4]x is under X's control at t if and only if the truth value of x is determined by X's choice in the context of a deliberation addressed by X at that time. I follow W.Spohn (1977, 1979) in disallowing the truth values of sentences to be optional for agents unless such sentences lack unconditional probabilities. (Thus, *Q*(x/T) where T is a logical truth or a sentence entailed by K is undefined because the truth value of T is not under X's current control.)

[5]*Q*(h/e) is a finitely additive and normalized probability measure in L relative to K if and only if *Q*(h/e) satisfies the following conditions.

(1) if Q(x/y) is defined relative to K, *Q*(x/y) ≥ 0.
(2) If K |- x ≡ x' and K |- y ≡ y', *Q*(x/y) =*Q*(x'/y').
(3) If K,z |- ~(x∧y), *Q*(x/z) + *Q*(y/z) = *Q*(x∨y/z). (Finite additivity)
(4) If K,y |- x, *Q*(x/y) = 1. (Normalization)
(5) *Q*(x∧y/z) = *Q*(x/y∧z)*Q*(y/z). (Multiplication theorem)
*Q*(x) is defined to be *Q*(x/T) for any sentence T entailed by K.

A *strictly Bayesian* confirmational commitment is one that satisfies confirmational conditionalization and the following condition:

*Credal Uniqueness*: If K is consistent, *C*(K) is a unit set.

Carnap introduced the notion of a *credibility* function representing an agent's disposition to change credal probability judgments upon the acquisition of new information via expansion. Carnap was a strict Bayesian so that he required credal uniqueness to be satisfied. Also, the credibility function was supposed to be a permanent disposition of the mature inquirer. On the view I have, it is not a disposition at all but rather a commitment or undertaking to institute changes in credal probability judgment with changes in the state of full belief (whether it be by expansion, contraction, replacement or residual shift). If the commitment were perfectly fulfilled, the agent would have a disposition to change credal probability judgments with changes in the state of full belief in the manner specified by the commitment. Finally, on the view I favor, in contrast to Carnap's, the commitment is subject to modification when there is good reason to do so.[6] Like states of full belief, confirmational commitments are corrigible.

**5. Probability Logic**

To be sure, if the choice of a confirmational commitment were to be dictated by a probability logic of some kind, the corrigibility of confirmational commitments would be called into question. But how shall we understand probability logic?[7]

The quasi Bayesian view suggests that the logical confirmational commitment is the weakest confirmational commitment allowed by the principles of confirmational or probability logic. Credal consistency, coherence and convexity along with confirmational conditionalization are the minimal requirements of a quasi Bayesian point of view.

Any confirmational commitment defined for language L satisfying these requirements can be characterized by identifying the logically weakest corpus UK expressible in L and specifying a convex set *C*(UK) of probability measures relative to UK. Every other corpus K under consideration is an expansion of UK by adding some sentence x. Confirmational conditionalization automatically determines the credal state for *C*(K).

The weakest confirmational commitment is the confirmational commitment *CIL* such that *CIL*(UK) is the largest convex set of credal probability measures relative to UK allowed by credal consistency, coherence and convexity together with any other constraints alleged to be

[6]Although Carnap (1962) did write that credibility functions are permanent dispositions of mature rational agents, he sometimes appeared to think that the confirmation functions that represented them are subject to change. The last part of (Carnap, 1952) is devoted to exploring ways of altering the adoption of confirmation functions with the availability of data. A more accurate description of Carnap's position suggests a certain ambivalence on the question of the revisability of credibility functions.
[7]Probability logic is Ramsey's term. Carnap used the expression "inductive logic". The same idea has also been expressed by the "logic of confirnmation".

mandatory on all confirmational commitments. There are familiar disputes concerning what such additional constraints might be. There is no consensus on what constitutes a complete probability or confirmational logic. My own view is that if there is a usable notion of objective statistical probability or chance, we must add a principle of direct inference linking full belief about chance with judgment of credal probability and that addition of this principle exhausts the resources of probability logic. If chance is dismissed as meaningless (as Savage and De Finetti do), then there is no additional principle. I am inclined to think that there are useful theoretical conceptions of chance and, hence, that probability logic should include principles of direct inference.

There are alternative quasi Bayesian views that endorse some form of insufficient reason or principle of maximum entropy. I agree with Ramsey, De Finetti and Savage, on the one hand, and with Venn, Peirce, von Mises, Kolmogoroff, et al. on the other that appeal to entropic notions of ignorance is not a helpful source of principles of probabilistic rationality.[8]

*Necessaritarians* insist that rational agents adopt the weakest confirmational commitment *CIL* allowed by probability logic. For necessaritarians, confirmational commitments are, indeed, incorrigible.

But unless probability logic is so powerful as to single out a standard probability function relative to <u>UK</u>, there will always be confirmational commitments stronger than the logical confirmational commitment. Probability logic will require rational agents to make probability judgments satisfying its requirements; but probability logic will not single out a confirmational commitment that everyone ought to adopt. As long as probability logic is satisfied, no restriction is placed on the confirmational commitment that may be adopted coherently.

According to the "where it doesn't itch don't scratch" [~I~S] principle, changes in confirmational commitment ought, like changes in states of full belief, to be implemented only when there is a good reason for doing so. *Revisionists* take the view than an inquirer should retain his confirmational commitment as long as there is no good reason for change. *Pace* Peter Gärdenfors, revisionists maintain that occasions for modifying confirmational commitments for good reason do arise and, indeed, may often do so.

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[8]Carnap seemed at one point to have hoped to identify a system of principles of probability logic sufficiently strong that exactly one probability measure would be permissible according to *CIL*(<u>UK</u>). A confirmation function is supposed to be a uniquely permissible credibility according to the principles of probability or inductive logic.

Skeptics about there being good reasons may complain. If a distinction between legitimate or justified changes in confirmational commitments and illegitimate ones is not recognized or if it is denied that there can be justified changes, endorsing [~**I**~S] implies that one is obliged after all to remain faithful to one's confirmational commitment just as necessitarians demand. The difference is that necessitarians insist on a common or standard confirmational commitment *CIL* whose authority is assured by probability logic. Let us call such a view *tenacious personalism*.

If probability logic permits rational agents to choose from a menu of alternative confirmational commitments, why should rational agent X be obliged to adopt a confirmational commitment and hold onto it indefinitely? Skeptics about justifying changes in probability judgment avoid the absurdity by rejecting the [~**I**~S] principle. *Personalists* who may be skeptical of the availability of good reasons for changing confirmational commitments will allow rational agents to change these commitments to others satisfying the requirements of probability logic as long as the changes are sincere and considered carefully.

*Tempered personalists* seek to give accounts of how confirmational commitments may be modified to accommodate the needs of the problems addressed in specific inquiries.[9] To this extent, they move closer to the views of revisionists. But insofar as the appeal to those features of the context they invoke fail to constrain probability judgment, tempered personalists become personalists.

One common kind of belief change occurs when X changes from corpus $\underline{K}$ to the expansion $\underline{K}^{+}_{x}$ and the credal state $\mathbf{B}_{x}$ relative to $\underline{K}^{+}_{x}$ is the conditionalization of **B** relative to $\underline{K}$. This kind of change is often called "conditionalization". I shall call it *temporal credal conditionalization*. The inverse change shall be called *inverse temporal credal conditionalization*.

The important thing to notice here is that as long as X retains the same quasi Bayesian confirmational commitment in changing from $\underline{K}$ to $\underline{K}^{+}_{x}$ or vice versa, temporal credal conditionalization and its inverse are mandated by the quasi Bayesian endorsement of confirmational conditionalization. And as long as $\underline{K} = \underline{K}^{+}_{x}$, there can be no change in credal state.

Quasi Bayesians who are necessitarians or tenacious personalists cannot, so it seems, allow for violations of temporal credal conditionalization or its inverse. Tempered personalists and revisionists can allow for such violations.

Recall that one of Gärdenfors's complaints concerning my representation of belief states by two components - a confirmational commitment and a corpus - derived from a worry that this approach could not provide for the representation of views that allow for violations of temporal credal conditionalization. Necessitarians and tenacious personalists rule out such representations. But tempered personalists and revisionists do not.

Gärdenfors alleges that decomposing doxastic states into states of full belief and confirmational commitments eliminates R.C.Jeffrey's rule for updating credal probabilities from consideration without any argument (Jeffrey, 1965). This is not true.

In Jeffrey updating the inquirer's state of credal probability judgment **B** relative to corpus K in language L changes to state of credal probability judgment **B**' relative to the same corpus K. There is, of course, more to be said about Jeffrey's account of updating. But this much alone suffices to establish that the change in credal probability judgment is representable as a change from confirmational commitment *C* to confirmational commitment *C′* such that *C*(K) = **B** and *C*'(K) = **B**'.

The change in confirmational commitment is of a special kind. A privileged set of hypotheses *P* exclusive and exhaustive relative to K is specified. Both the initial confirmational commitment *C* and the confirmational commitment *C′* to which it is altered are strict Bayesian. Hence, they are quasi Bayesian and thus conform to confirmational conditionalization. Moreover $C(K^+_x) = C'(K^+_x)$ for every hypothesis x in the set *P*. We shall say that *C* and *C′* satisfy the "rigidity conditions" relative to *P* (Jeffrey, 1970).

The set of uniquely permissible credal probability distributions over the members of *P* according to *C*(K) is different from the set of uniquely permissible distributions over the members of *P* according to *C′*(K). Thus, the shift from *C* to *C′* corresponds to a change in the set of uniquely permissible distributions over the members of *P*.

According to Jeffrey, this change is caused by sensory stimulation.

Whatever one might think of Jeffrey updating, it is not ruled out of consideration when doxastic states are decomposed into states of full belief and confirmational commitments. To the contrary, Jeffrey updating is seen as a species of change in doxastic state where the state of full belief remains fixed and the confirmational commitment is altered in a manner satisfying rigidity conditions. There are many modes of change in confirmational commitments that satisfy the rigidity conditions relative to a privileged partition. Most of them differ from Jeffrey updating in that the change in the credal distribution over *P* is not required to be caused by sensory stimulation.

Suppose X knows that weight is normally distributed with unit standard deviation. The privileged partition *P* specifies values of the mean of the normal distribution. This partition is determined by the problem under investigation and the space of potential answers or solutions

[9]A. Shimony (1970) is an excellent expression of what Shimony himself labels "tempered personalism".

recognized by X. X's demands for information and not the causal origins of X's beliefs control the choice of *P*.

X might begin with a confirmational commitment that recognizes every coherent distribution over the values of the mean to be permissible. It is maximally indeterminate. X might shift subsequently to a confirmational commitment that recognizes a convex subset of the distributions over *P* to be permissible but keeps the confirmational commitment intact with respect to expansions of K by adding elements of *P* to K.

Typically the problem X addresses is that of choosing a prior probability distribution over the parameter space *P*. I suggest representing such a problem as beginning with a maximally indeterminate prior where all probability distributions over *P* are permissible. Choosing a prior then becomes choosing a more determinate set of probability distributions.

This change in confirmational commitment is not strictly Bayesian because of the indeterminacy in the initial set of distributions over *P*. For those whose gospel is strictly Bayesian, let the initial distribution over *P* be maximally determinate and let X be dissatisfied with it. Let the subsequent distribution be maximally determinate as well. Formally the change simulates Jeffrey updating. The rationale for implementing the change and, perhaps, the causal mechanisms differ.

No matter which variant of this situation is considered, a common problem for Bayesian statisticians has been identified. How does one rationalize shifts in confirmational commitment involved in choosing a prior distribution over a parameter space? The modeling of the problem is typically done so as to insure that the rigidity conditions are satisfied. Like Jeffrey updating a change in confirmational commitment is "propogated" by changing the set of permissible distributions over a partition *P* that the context somehow privileges.

The difference from Jeffrey updating resides in how the initiating change in the set of permissible prior distributions is rationalized or, if not rationalized, explained. Assuming that the inquirer restricts his confirmational commitments to quasi Bayesian confirmational commitments obeying confirmational conditionalization, the background information and data initially available cannot decide between the rival confirmational commitments. But the way the problem under investigation has been formulated so as to bring out the kind of information being sought can be used to select a prior that will not bias the conclusions reached after the outcomes of experiments have been ascertained (Levi, 1980, 13.4).

Jeffrey updating by way of contrast is equivalent to changing the initial quasi Bayesian confirmational commitment to another quasi Bayesian confirmational commitment in a way that is responsive to sensory inputs that do not lead to the acquisition of full belief in the truth of new information.

If a witness has only a hazy impression of the color of a car involved in an accident, the witness report is generally ignored in determining what the color of the car was. The haziness of the report is a mark of its unreliability.

But perhaps some information can be squeezed out of the witness's responses. We should not come to fully believe that the color of the car is red just because the witness said so. Perhaps, however, we should come to judge the probability that the car is red to be 0.85. When we are changing our probability judgments in the light of our own observations of the color of the car, advocates of Jeffrey updating sometimes seem to suggest that we are treating our own experience just like the witness's.

One might think that this process is nothing more than conditionalizing on the data (that is fully believed) that the witness testified that the color of the car is red. In the case of our own observations, we conditionalize on reports of how the color seems to us. Jeffrey himself did not seem to have this understanding in mind. Presumably we have no information about how the color seems to us. We just respond directly to sensory stimulation by changing our probability judgments. Jeffrey updating does not seek to exploit information carried in reports of witnesses or our own reports. It seeks to exploit information allegedly carried by sensory inputs that we do not or cannot characterize. Instead of urging that we try to do better, Jeffrey proposed a way mode of changing confirmational commitments that is allegedly sensitive to such sensory input.

We face the following dilemma. If X acquires no information as to the source of the initiating change in sensory experience or acquires no information on the basis of which the initial distribution over *P* is altered, X cannot engage in subjecting the modification of his own views to the relevant kind of critical scrutiny. But if he can acquire such relevant information, then the change in the initial distribution over *P* may well be the product of conditionalization. No change in confirmational commitment need occur.

To be sure, there may be a pseudo change in confirmational commitment. The language used to represent states of full belief may not be rich enough to express the information on the basis of which the distribution over *P* us modified by conditionalization. The appearance of a change in confirmational commitment may well be a product of a failure to invoke a rich enough linguistic apparatus for representing the changes in view that are involved.

In sum, Jeffrey conditionalization may be nothing more than temporal credal conditionalization when a richer linguistic apparatus is deployed. Or it may reflect a change in confirmational commitment while remaining faithful to confirmational conditionalization and the rigidity condition. That there are such changes is clear from the problem of choosing priors. Jeffrey updating is not a case of choosing priors. That much is clear. Very little else about it is.

Jeffrey himself seems to have backed off his initial interest with prescriptive standards for justifying changes in credal probability judgment over time. Radical probabilism makes no distinction between credal states at different times. As long as they are coherent with the requirements of the calculus of probability, they are acceptable. Radical probabilism is a species of personalism as I have characterized it.

My concern here, however, is not to address the bizarre ins and outs of radical probabilism. Peter Gärdenfors does not appear to be a radical probabilist. But he seems to understand his own pioneering work on belief change within a framework that retains some elements of a flawed probabilism.

Full belief is not distinguished from credal probability 1 and change in state of full belief is not separable from change in probability judgment. In spite of this Gärdenfors has made pathbreaking contributions to the study of changes in states of belief (that appear to be changes in states of full belief). It seems, however, that he must understand these contributions as first approximations due to his abstracting away from the quantitative aspects of belief change. More precise characterizations represent belief states by sets of probability distributions as Gärdenfors and Sahlin (1982) have done. Belief sets as Gärdenfors understands them would then be those sentences or propositions that are assigned probability 1 according to all probability functions in the set.

In this discussion, I have sought to offer some reasons for doubting this view. A state of full belief is not a qualitative abstraction from a precisely specified doxastic state. It is one *component* of a doxastic state. The other is the confirmational commitment. I have argued that by regarding probability judgment under the guise of confirmational commitment and state of full belief as separable components of an inquirer's doxastic commitments at a time, one can obtain a comprehensive purchase on all aspects of belief change with minimal question begging.

Thus, students of belief revision in the style of AGM may explore this topic without examining how judgments of probability are constrained by belief changes. Yet, it is also possible to study changes in credal probability judgment when states of full belief are altered by expansion, contraction, and various kinds of revision while the confirmational commitment remains constant.

It also becomes possible to explore the strengthening (expansion) and weakening (contraction) of confirmational commitments while the state of full belief remains fixed and when it is allowed to vary in diverse ways.

To my way of thinking, the separation of doxastic states into two components in the manner indicated offers promise of a rich account of belief change that Peter's suggestion does. It does so along lines congenial with ideas that Peter himself pioneered with such distinction

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